Regularity properties of parallel volume and surface area

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Given a compact set A in R^d, the r-parallel sets A_r are a particularly nice way to approximate A (as the parallel radius r tends to 0), encoding much of the geometry of A. They are the key to many geometric objects like curvatures measures, Minkowski contents and geometric zeta functions. It is well known that the volume function of A (associating to r the volume of A_r) is differentiable at all $r > 0$ except countably many and that its derivative is related to the surface area of A_r. We discuss localizations of this result and consequences. In particular, we show that at differentiability points $s > 0$ of the volume function, the surface area measures of r-parallel sets of A converge weakly to the surface area measure of the s-parallel set as $r \to s$.

We also study the question which (countable) sets of parallel radii are possible as sets of non-differentiability points of the volume function of some compact set. We provide a full characterization for dimensions $d = 1$ and 2.

The latest results are based on joint work with Jan Rataj.